Why Mathematical Propositions are *Synthetic,* not *Analytic* Judgments

Kant's Reasons

First: Remember that in *general* a judgment is any proposition/assertion that takes the form "A is B" and means *either* that B is being *predicated* of A (e.g., "The Ball [A] is red [B]") **or** means "A is identical to B" (e.g., "Jason is Jason").

Second: Kant makes the distinction between *Analytic* judgments and *Synthetic* judgments as follows:

An **Analytic Judgment** "… [is one] in which the relation of a subject [A] to the predicate [B] is thought …, this relation is possible in two different ways….[in which case] the predicate *B* belongs to the subject *A* as something that is (covertly) contained in this concept A [in which case it is an *analytic* judgment]…" (A6/B10, *CPR*)

> JP: In these judgments, all one needs to do in order to establish that an *analytic relation* exists between the concept of the subject A and the predicate B is ***to analyze the concept of the subject A*.** If this **reveals/uncovers the concept B** then the judgment is analytic.

A **Synthetic Judgment** "… [is one] in which the relation of a subject [A] to the predicate [B] is thought …, [where the predicate] *B* lies entirely outside the concept *A*, though to be sure it stands in connection with it."

JP: In these judgments, something must ***join*** the concept of the subject *A* with the concept of the predicate *B*, and for Kant there is only one legitimate way that can occur: **through a Synthesis in the Manifold of a Pure or Empirical Intuition.**

Proof that Arithmetical Judgments are Synthetic

The proof relies on showing that such judgments **cannot be analytic**. Since all judgments must be *analytic* **or** *synthetic*, if a type of judgment *cannot be analytic* then it *must be synthetic.*

Consider the arithmetic judgment "7 + 5 = 12"

First, invert the order of the equation (permissible as such inversions do not change the meaning of the judgment) to read "12 = 7 + 5"

This judgment has the form "A [12] is B [7 + 5]". It therefore means "A [12] is identical to B [7 + 5]" Clearly this is *true*, and *necessarily so*. If it is analytic, then the concept "12" *contains* the concept "7 + 5". Let's agree, for the sake of the argument, that this is the *correct* way to understand what assertion of the judgment means, and so accept the claim that the concept "12" *does contain* the complex concept "7 + 5".

But if this is true, then here is a list of *other* arithmetic judgments that are *also true:*

"12 = 13 − 1"; "12 = 2 x 6"; "12 = 1,000,024 − 12"; "12 = 24,000,000 ÷ 2,000,000"; and ***infinitely many OTHER arithmetic judgments*.**

But if the concept of A contains *infinitely many complex concepts* B, that would suggest that the concept of A is infinitary. **But it is not**.

Therefore, "12 = 7 + 5" is NOT an analytic judgment. SO it must be synthetic.

Furthermore: in *grasping* each of the judgments that are *equivalent* to "12 = 7 + 5", the content of the *act of thought* that generates the judgment is different in each of the infinitely many equivalent judgments! (and this is something that Kant's view of all judgments requires: a judgment is simply the act of *thinking* one concept as *falling under another* concept).

Proof that Arithmetic Judgments are Synthetic *A Priori* Judgments

Now that we know “7 + 5 = 12” is a synthetic judgment, we can then consider that it is also (along with all of the infinitely many *seemingly equivalent* judgements) a ***necessarily true judgment***. But that means it cannot be an *a posteriori* judgment because all of those depend on the contingent information made available through *empirical intuitions*, and **all such judgments are Synthetic A Posteriori!**

Thus we have proved that “7 + 5 = 12” and all other arithmetic judgments are Synthetic A Priori judgments.

BONUS: we have now also proved AGAIN that synthetic a priori judgments are possible (since any actual true judgments are thereby evidence that such judgments are *possible!*)

Proof that Arithmetic Judgments Presuppose the Pure Intuition of Time

Premise 1: All synthetic judgments can only be justified by appeal to a synthesis in the manifold of an intuition.
Premise 2: No empirical intuition can establish a *necessarily true judgment*.
Premise 3: Time, as the pure form of inner and outer sense empirical intuitions (and thus, one of the *conditions of the possibility of such intuitions*), permits the generation of a synthesis of moments of time *in a pure intuition* that contains none of the *matter of intuition* (sensations or self-affections).
Premise 4: (3) allows a mind like ours to produce a pure representation of any whole number as a *series of moments in time*, and this constitutes a *pure intuition of that series* of moments.
Premise 5: (4) Means that “7” and “5” can be represented as two series of pure (nonempirical) moments in time, and these can then be *represented to consciousness together by means of the very synthesis that makes it possible to represent each of the numbers as series of moments in time.* When that new, pure intuition of 7 moments in time combined with 5 moments of time is then apprehended in the resulting *series of moments in time*, it **displays in a pure intuition that 7 and 5, when combined**, **equal 12**.
Conclusion: Given (5), the justification of the necessary truth “7 + 5 = 12” presupposes time as the pure form of inner and outer sense respectively.

Side benefit: This **proves** that in the domain of perception, wherever you find anything that can be represented by the numbers 7 and 5, you *know* that what is represented *must constitute a representation of the number 12.* This establishes that **arithmetic propositions** necessarily and justifiably applicable to the world of our sense perceptions (all of which are governed by the pure form of time).

Additional benefit: Transcendental Idealism received further support from this result.

Proof that Geometrical Judgments are Synthetic

Premise 1: Assume that “A” (my left hand) and “B” (my right hand) are represent ‘perfect physical counterparts’.
Premise 2: The concept of “A” is different from the concept of “B”
Premise 3: Given (2), necessarily “A = B” is not analytically true
Premise 4: “A is not equal to B” is true.
Premise 5: All true judgments are either analytic or synthetic.
Premise 6: But “A is not equal to B” is *also* not analytically true (the difference in their concepts cannot be represented except through a feature of space that includes *orientation* in space [left/right; top/bottom; etc.])
Interim Conclusion: “A is not equal to B” requires a representation in a synthesis in intuition.
Premise 7: “A is not equal to B” is necessary true.
Interim Conclusion: Given (7), “A is not equal to B” can only be represented by a *pure intuition*.
Premise 8: The pure intuition that can represent the difference between A and B is *the pure intuition of space.*
Conclusion: “A is not equal to B” presupposes, for its justification, the pure intuition of space.

Benefit: Judgments in geometry like the one given in this argument **are synthetic a priori.**

Additional benefit: Transcendental Idealism receives further support from this result.